Reduction of Variability in Split–Merge Systems

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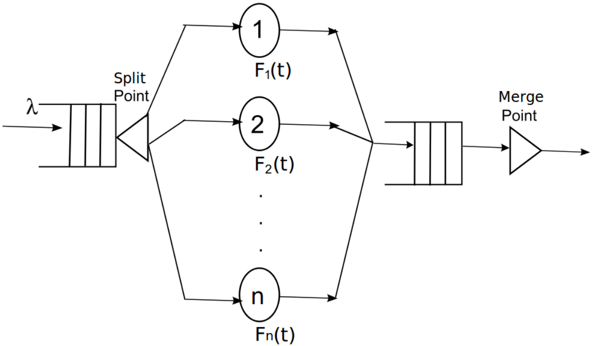
September 29, 2011

Introduction



- Proposition is to reduce variability
- Minimise utilisation of physical buffer space

- Vector of Delays $\mathbf{d} = (d_1, d_2, ..., d_i, ..., d_{n-1}, d_n)$

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- Cost Function $C(\mathbf{d}) = E(X) - E(Y)$, where X and Y are random variables denoting max and min completion time across all items in an order

$$F_X(t) \sim \prod_{i=1}^{n} F_i(t - d_i) \tag{1}$$

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$$\widetilde{\mathbf{d}} = (\widetilde{d}_1, \widetilde{d}_2, ..., \widetilde{d}_{i-1}, 0, \widetilde{d}_{i+1}, ..., \widetilde{d}_{n-1}, \widetilde{d}_n) \tag{5}$$

Numerical Results

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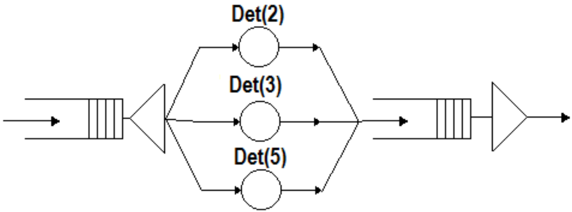

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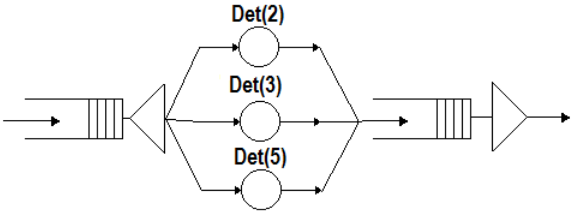
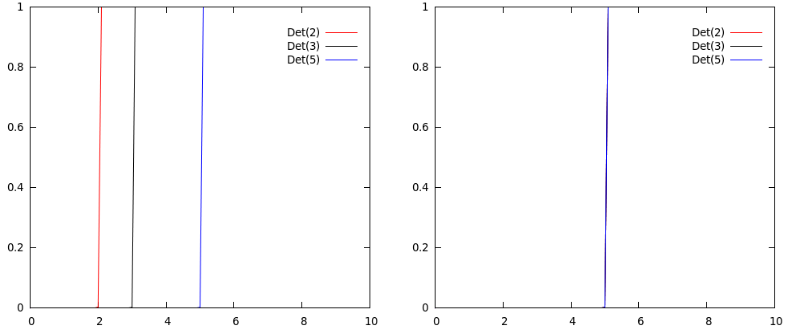
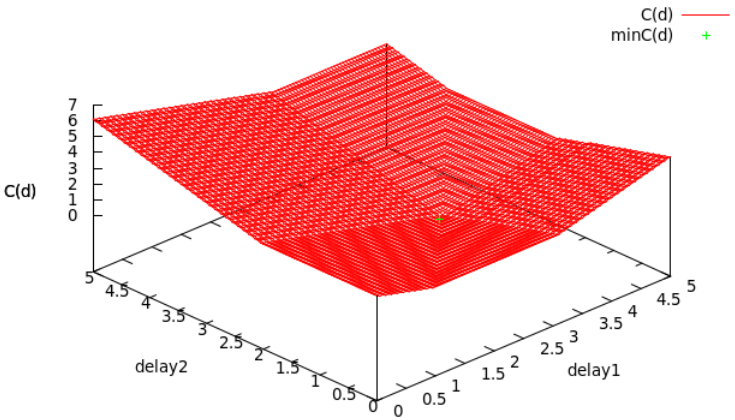


3. Vector of optimal delays $\widetilde{\mathbf{d}} = (3.0, 2.0, 0.0)$

CDFs of server service times before and after adding the optimal delays



Surface plot of cost function against delays



Numerical Results (non-trivial case)

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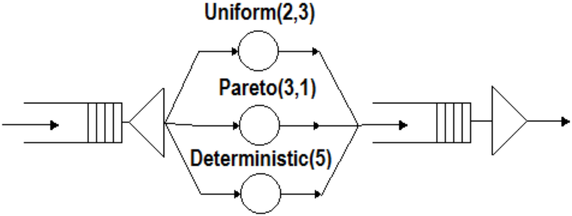

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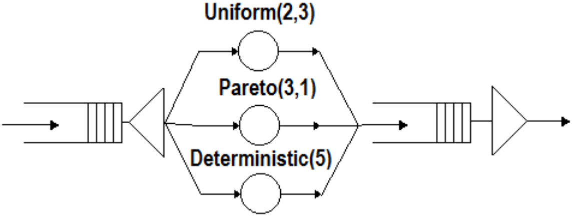

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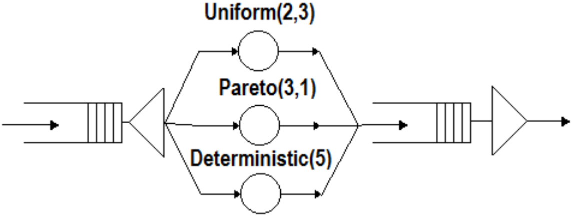
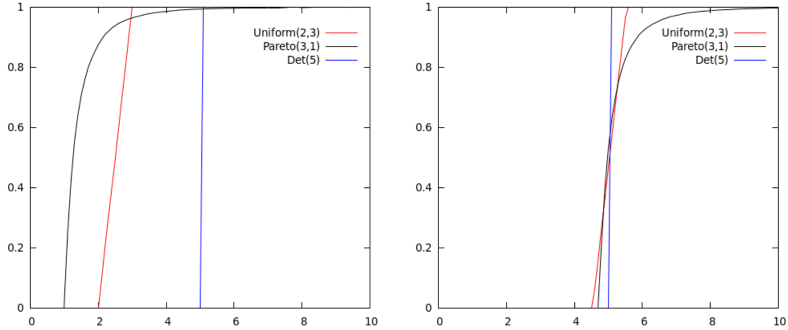


3. Vector of optimal delays $\widetilde{\mathbf{d}} = (2.5317, 3.7154, 0.0)$
4. Intuitively expected delays $\mathbf{d}_{intuit} = (2.5, 3.5, 0.0)$, $\widetilde{\mathbf{d}} \neq \mathbf{d}_{intuit}$

CDFs of server service times before and after adding the optimal delays



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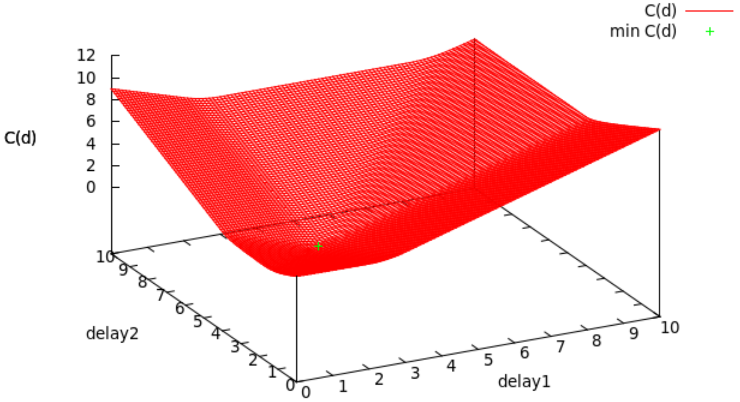

Future Work

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3. Investigate analogous optimisation of a fork-join system.

Thank you!